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ABSTRACT

The purpose of this study was: (a) to develop a learning hierarchy for rational number subtraction using intraconcept analysis, and (b) to validate that hierarchy using the Walbesser technique and pattern analysis. Skills required to complete tasks within the hierarchy were operationally defined and ordered from both a mathematical and psychological point of view. A test utilizing an item for each level in the hierarchy was administered to a sample of 200 children in grades four through eight. The final hierarchy generated and important implications for prescriptive instruction relative to subtraction of rationals are discussed. The specific tasks used in the study are appended. The initial hypothesized ordering of these tasks and the empirically determined hierarchy are presented. (Author/SD)

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An Intraconcept Analysis of Rational Number Subtraction: A Validation Study

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In previous studies, researchers have attempted to generate learning hierarchies using task analysis based primarily on epistemological considerations (Gagne & Paradise, 1961; Gagne, 1962; Cox & Graham, 1966; Uprichard, 1970; Okey & Gagne, 1970; Harke, 1971; Riban, 1971; Phillips & Kane, 1973; Miller & Phillips, 1975). Studies of this type conducted in the early sixties provide substantial evidence to support the hierarchical structure of knowledge (Gagne & Paradise, 1961; Gagne & Brown, 1961; Gagne, 1962, 1963; Gagne, Mayor, Garstens & Paradise, 1962; Gagne & Staff, 1965). An examination of results from recent studies (Neidermeyer, Brown & Sulzen, 1969; Brown, 1970; Phillips & Kane, 1973; Callahan & Robinson, 1973) suggests that optimal learning sequences can be developed by sequencing instructional materials according to validated learning hierarchies. However, both Gagne (1968) and Pyatte (1969) have pointed out that the determination of an optimal or hierarchical sequence of subtasks from simplest to most complex is not easily achieved.

Numerous hierarchy validation techniques have appeared in the literature. Critical analyses of the efficacy of these techniques are also reported (Resnick & Wang, 1969; Eisenberg & Walbesser, 1971; White, 1973; White, 1974a, 1974b; Phillips, 1971, 1974). Many of these techniques are concerned

020 892

ERIC

with the analysis of data collected on hypothesized hierarchies. While there is considerable room for improvement, techniques such as the Walbesser Method (Walbesser, 1968) and Pattern Analysis (Rimoldi & Grib, 1960) have been used effectively in validating hypothesized hierarchies (Harke, 1971; Phillips & Kane, 1973; Miller & Phillips, 1975).

The investigators in the present study were concerned with the generation and validation of an hypothesized learning hierarchy. More specifically, the purposes of the study were: (1) to develop a learning hierarchy for rational number subtraction using the intraconcept analysis technique (Uprichard and Phillips, 1975), and (2) to test the validity of the hypothesized hierarchy using the Walbesser Technique and Pattern Analysis. The objective in doing an intraconcept analysis is to generate a series of tasks, each of which represent an operationalized level of a given concept. This technique differs from the more widely used task analysis model in that consideration is given to both psychological and content (discipline) factors in developing and hierarchically ordering tasks. Because of this, the authenticity of many tasks generated using the intraconcept analysis technique may be questioned by content area specialists. The present study, the second in series, was conducted in hopes of shedding further light on the epistemological-psychological balance needed in developing efficient or optimal instructional sequences in mathematics.[1]

[1]The first study, "An Intraconcept Analysis of Rational Number Addition" was presented at AERA 1975.

PROCEDURE

Development of Hierarchy

The intraconcept analysis strategy used to analyze the subtraction of rationals in fraction form was as follows:

a) Problems were divided into two levels - like denominators, and unlike denominators. Classes within each level were identified by the nature of (prime and composite) and relationship between (multiple or not) denominators of the two rationals subtracted. The classes, in their hypothesized order of difficulty, were:

Like Denominators

A. Composites. (2/4 - 1/4 = ___)

B. Primes. (2/5 - 1/5 = ___)

Unlike Denominators

C. Composites - one composite multiple of other. (1/4 - 1/8 = ___)

D. Prime, composite - composite a multiple of prime. (1/5 - 1/10 = ___)

E. Primes. (1/5 - 1/7 = ___)

F. Prime, composite - composite not multiple of prime. (1/5 - 1/9 = ___)

G. Composites - not multiples. (1/6 - 1/8 = ___)

b) Within each denominator class an attempt was made to generate five renaming categories. The renaming categories were operationally defined in terms of

ERIC

renamings required in the difference and/or sum (minuend).[2]

I. Difference - proper fraction in simplest form without renaming. (2/4 - 1/4 = 1/4)

II. Difference - proper fraction renamed to zero. (1/4 - 1/4 = 0/4 = 0)

III. Difference - proper fraction renamed to simplest form. (3/4 - 1/4 = 2/4 = 1/2)

IV. Renaming in sum, difference - proper fraction in simplest form without renaming. (1 1/4 - 2/4 = 5/4 - 2/4 = 3/4)

V. Renaming in sum, difference - proper fraction renamed to simplest form. (1 1/4 - 3/4 = 5/4 - 3/4 = 2/4 = 1/2)

c. Two tasks involving mixed numerals (whole numbers greater than one and less than ten) were associated with each renaming category within a denominator class. A renaming category and its two related tasks is presented below:

Renaming Category I	Task # 1	Task #2
2/4 - 1/4 = ___	3 2/4 - 1 1/4 = ___	3 2/4 - 1/4 = ___

d. Nine additional subtraction problems with whole number sums were included in the hierarchy. These problems are:

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[2]Five renaming categories do not exist in some denominator classes.

$1 - 1/4 = 3/4$ $\quad$ $3 - 1\ 1/4 = 2\ 4/4 - 1\ 1/4 = 1\ 3/4$

$3 - 1/4 = 2\ 4/4 - 1/4 = 2\ 3/4$

$1 - 2/4 = 2/4 = 1/2$ $\quad$ $3 - 1\ 2/4 = 2\ 4/4 - 1\ 2/4 = 1\ 2/4 = 1\ 1/2$

$3 - 2/4 = 2\ 4/4 - 2/4 = 2\ 2/4 = 2\ 1/2$

$1 - 2/3 = 1/3$ $\quad$ $3 - 1\ 2/3 = 2\ 3/3 - 1\ 2/3 = 1\ 1/3$

Analysis of Hierarchy

The above procedures yielded ninety-six specific tasks. In order to hold testing to a manageable size, some tasks within classes were collapsed. The final hierarchy consisted of 47 tasks. Based upon the hypothesized ordering of the subordinate tasks, a test was constructed to assess mastery at each level in the hierarchy. "Pass" was defined as correct difference expressed in simplest form. For ease in computation the tasks appeared in vertical form. The test was administered to 200 students in grades 4 through 8 in order to obtain a wide range of ability levels. The majority of the Ss were in grades 5 and 6. Subjects who passed or failed all 47 items were excluded from the study since inferences about the order of items cannot be based on responses from these students (Phillips & Kane, 1975). The resulting sample contained 174 Ss.

The patterns of responses for each transfer in the hierarchy were analyzed using both the Walbesser Technique (Walbesser, 1968) and Pattern Analysis (Rimoldi & Grib, 1960). The Walbesser Technique is based on the 2 X 2 contingency table of pass-fail responses (Figure 1).

		Item 1	
		fail (-)	pass (+)
Item 2	pass (+)	-+	++
	fail (-)	--	+-

Figure 1. Contingency Table

Using the frequency of students falling in each cell above, the following three ratios were computed for every possible relationship in the hierarchy (i.e. level 1 with 2, 1 with 3, ... 1 with 47; 2 with 3, 2 with 4, ... 2 with 47, etc.).

(1) Consistency Ratio $= \dfrac{(++)}{(++) + (+-)}$

The value of this ratio is a measure of how consistent the data are with the hypothesized dependency.

(2) Adequacy Ratio $= \dfrac{(++)}{(++) + (--)}$

The value of this ratio is a measure of the adequacy of the identified levels.

(3) Completeness Ratio $= \dfrac{(++)}{(++) + (--)}$

The value of this ratio is a measure of the effectiveness of instruction.

The level of acceptability used for each of these ratios was that determined by Phillips (1971) instead of those proposed by Walbesser since no instructional sequences were involved.

These levels are: (1) Consistency Ratio .85, (2) Adequacy Ratio .70, (3) Completeness Ratio .50.

The pattern analysis technique was used to analyze the responses for the complete hierarchy on a subject by subject basis. The index of agreement given by the pattern analysis indicates the amount of agreement or correlation between two patterns. In this case, the index of agreement indicates the agreement between the observed and expected patterns. If the tasks were truly hierarchical, where each subtask was a necessary prerequisite to the next, once a learner failed a given level he would be expected to fail all subsequent levels. Thus, the expected pattern was defined as one where no correct responses followed an incorrect response.

RESULTS

The initial hypothesized hierarchy developed using an intraconcept analysis is given in Table 1. A computer program based on the Walbesser Technique was used to give the pass-fail

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Insert Table 1 About Here

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response patterns between all relationships. That is, item 1 was paired with all 47 items; item 2 with all items; etc.; until all possible pairs of items were onsidered.

In order to analyze the hypothesized hierarchical sequence, the consistency, adequacy, and completeness ratios for each relationship within the hierarchy were examined. The ordering

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which yielded the best fit to the data is shown in Table 2.

Insert Table 2 About Here

The pattern of responses obtained from the hypothesized ordering of subordinate tasks yielded an index of agreement of .63. After final revision, the empirical sequence yielded an index of agreement of .75 which indicates a higher agreement between the expected and observed response patterns. No statistical test of significance for the index of agreement has been developed.

The internal consistency of the test based on the initial hierarchy was determined using the Kuder-Richardson Formula 20 (Nunnally, 1967). The value of this coefficient was .96. The pattern of responses for the empirical sequence was analyzed to determine if the ordering exhibited a hierarchical structure based on item difficulty. Item difficulty of each task in the ordered sequence is given in Table 3. Although

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Insert Table 3 About Here

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there would be some discrepancies between a hierarchical sequence based on item difficulty and the ordered sequence determined, the general pattern of item difficulty of the latter is acceptable. Further, item difficulty alone is not considered an adequate technique for determining hierarchical relationships (Phillips, 1971).

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DISCUSSION AND IMPLICATIONS

An examination of the data in Table 2 (Walbesser ratios) indicates that with few exceptions task comparisons within the empirically determined hierarchy meet specified criteria on two of the three ratios. The consistency ratio was met in 34 of 46 comparisons, the Adequacy Ratio in 45 of 46, and the Completeness Ratio in 43 of 46. It should be noted that of the eleven comparisons not meeting the criteria level for the Consistency Ratio, five missed the criteria by .02 or less. The index of agreement for the hypothesized forty-seven task hierarchy was .63. After the tasks were rearranged in an empirical hierarchy based on Walbesser ratios the index of agreement was .75.

The empirically determined sequence is presented in Table 4. An examination of the sequence tends to support the following implications for teaching youngsters the subtraction of rationals in fraction form.

Insert Table 4 About Here

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1. In subtracting rational numbers in fractional form, finding least common denominators is not as difficult for learners as the renaming(s) (sum and/or difference) required.
2. Within renaming categories I and II, tasks involving like denominators (3/4 - 2/4 = ___; 3 1/2 - 2 2/4 = ___;

3/4 - 3/4 = ___; 3 3/4 - 3/4 = ___) should be taught before proceding to tasks involving unlike denominators (1/2 - 1/3 = ___; 3 1/2 - 2 2/4 = ___; 4/8 - 2/4 = ___; 3 4/8 - 2/4 = ___).

3. Within renaming categories I and II, tasks involving unlike denominators (1/3 - 1/4 = ___; 4/8 - 2/4 = ___) should be taught before those involving like denominators in renaming category III (3/6 - 1/6 = ___).

4. Within renaming category III, tasks involving unlike denominators (5/12 - 1/6 = ___) should be taught before those involving like denominators in renaming categories IV and V (1 1/4 - 2/4 = ___; 1 1/6 - 3/6 = ___).

5. Within a renaming category, subtraction of two mixed numerals with whole number parts greater than one (6 1/6 - 3 2/4 = ___) should be taught before subtraction involving one mixed numeral (7 3/6 - 3/4 = ___).

6. Tasks involving whole number sums greater than one (5 - 1/6 = ___) should be taught after work is completed with like denominators in renaming categories IV and V (1 1/4 - 2/4 = ___; 1 1/7 - 2/7 = ___; 1 1/6 - 3/6 = ___).

7. Tasks involving whole number sums (greater than one) requiring renaming in the difference (6 - 1 2/10 = ___; 5 - 2/4 = ___) should be taught before those

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which require no such renaming (3 - 1 1/4 = ___; 4 - 1/6 = ___).

8. Within renaming categories I, II, III the hypothesized ordering of unlike denominator classes (C, D, E, F, G) was verified. However no pattern emerges for sequencing denominator classes in renaming categories IV and V.

9. Sequencing of tasks in renaming categories IV and V is difficult. This may be a result of renaming interactions between sums and differences.

In general, the results of this study, though limited to the subtraction of rational numbers in fraction form, support the notion that both epistemological and psychological factors be considered when developing teaching sequences in mathematics. Some of the implications above would not necessarily be derived from logical analysis alone. Also, in interpreting the results of this study one must be conscientious of the limitations of indirect validation procedures. For example, confounding variables such as prior educational experience of subjects and errors of measurement must be considered.

TABLE 1

Initial Hypothesized Hierarchy

1. 3/4 - 2/4 = ___	25. 5/12 - 1/6 = ___
2. 2/3 - 1/3 = ___	26. 2/5 - 2/10 = ___
3. 5/8 - 5/8 = ___	27. 6/9 - 1/2 = ___
4. 2/5 - 2/5 = ___	28. 3/4 - 3/6 = ___
5. 3/6 - 1/6 = ___	29. 3 1/8 - 2 1/16 = ___
6. 3 2/6 - 2 1/6 = ___	30. 6 3/9 - 3 1/3 = ___
7. 3 4/7 - 2 3/7 = ___	31. 8 4/8 - 2 1/3 = ___
8. 1 - 1/8 = ___	32. 4 1/6 - 1/12 = ___
9. 1 - 2/6 = ___	33. 3 1/5 - 2/10 = ___
10. 4 - 1/6 = ___	34. 5 3/5 - 3/9 = ___
11. 5 - 2/4 = ___	35. 1 1/8 - 1/4 = ___
12. 3 - 1 1/4 = ___	36. 1 1/9 - 1/3 = ___
13. 6 - 1 2/10 = ___	37. 1 1/5 - 2/3 = ___
14. 1 1/4 - 2/4 = ___	38. 1 2/5 - 4/9 = ___
15. 1 1/7 - 2/7 = ___	39. 1 1/8 - 1/6 = ___
16. 1 1/6 - 3/6 = ___	40. 1 3/18 - 3/9 = ___
17. 1/4 - 1/8 = ___	41. 1 1/5 - 4/10 = ___
18. 1/3 - 1/9 = ___	42. 1 6/9 - 4/5 = ___
19. 1/2 - 1/3 = ___	43. 1 2/6 - 6/9 = ___
20. 1/3 - 1/8 = ___	44. 4 1/5 - 2 2/7 = ___
21. 1/6 - 1/8 = ___	45. 6 1/6 - 3 2/4 = ___
22. 1/8 - 2/16 = ___	46. 2 1/3 - 2/5 = ___
23. 4/14 - 2/7 = ___	47. 7 3/6 - 3/4 = ___
24. 5/10 - 4/8 = ___	

TABLE 2

Walbesser's Ratios For The Empirical Ordering (N = 174)

Level	Consistency	Adequacy	Completeness
1-2	.99	.99	1.00
2-3	.99	.97	1.00
3-4	1.00	1.00	.97
4-6	.97	.91	.99
6-7	.94	.99	.96
7-8	.98	.87	.97
8-17	.90	.97	.95
17-18	.97	.95	.95
18-19	.95	1.00	.95
19-20	1.00	.96	.95
20-21	.97	.94	.94
21-22	.94	.92	.93
22-23	.99	.97	.88
23-24	.97	.94	.87
24-29	.90	.90	.90
29-30	.94	.93	.86
30-32	.94	.94	.86
32-33	.96	.92	.83
33-5	.83	.85	.86
5-25	.89	.85	.81
25-26	.84	.87	.81
26-27	.89	.83	.79
27-28	.88	.92	.76
28-14	.84	.84	.81
14-15	.92	.94	.78
15-16	.96	.89	.76
16-35	.89	.90	.75
35-36	.93	.92	.74
36-37	.89	.87	.74
37-9	.83	.84	.74
9-12	.88	.71	.66
12-10	.95	.75	.67
10-13	.94	.72	.66
13-11	.89	.75	.62
11-31	.80	.79	.69
31-39	.84	.79	.64
39-41	.85	.85	.62
41-43	.89	.72	.55
43-44	.69	.79	.52
44-38	.83	.78	.59
38-45	.86	.77	.58
45-46	.77	.88	.58
46-47	.90	.71	.55
47-40	.70	.72	.46
40-34	.57	.76	.36
34-42	.59	.44	.21

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TABLE 3

Item Difficulty - Empirical Hierarchy (47 Tasks)

Task	Difficulty	Task	Difficulty
1 (Simplest)	1.00	14	.74
2	1.00	15	.75
3	.98	16	.70
4	.98	35	.72
6	.93	36	.71
7	.97	37	.70
8	.85	9	.69
17	.93	12	.53
18	.93	10	.72
19	.96	13	.54
20	.93	11	.69
21	.91	31	.62
22	.88	39	.61
23	.87	41	.62
24	.85	43	.48
29	.83	44	.58
30	.81	38	.57
32	.80	45	.55
33	.81	46	.61
5	.78	47	.48
25	.79	40	.48
26	.78	34	.35
27	.72	42 (Most Difficult)	.31
28	.75		

TABLE 4

Empirical Hierarchy of Tasks

1. 3/4 - 2/4 = ___ (Simplest)

2. 2/3 - 1/3 = ___

3. 5/8 - 5/8 = ___

4. 2/5 - 2/5 = ___

6. 3 2/6 - 2 1/6 = ___

7. 3 4/7 - 2 3/7 = ___

8. 1 - 1/8 = ___

17. 1/4 - 1/8 = ___

18. 1/3 - 1/9 = ___

19. 1/2 - 1/3 = ___

20. 1/3 - 1/8 = ___

21. 1/6 - 1/8 = ___

22. 1/8 - 2/16 = ___

23. 4/14 - 2/7 = ___

24. 5/10 - 4/8 = ___

29. 3 1/8 - 2 1/16 = ___

30. 6 3/9 - 3 1/3 = ___

32. 4 1/6 - 1/12 = ___

33. 3 1/5 - 2/10 = ___

5. 3/6 - 1/6 = ___

25. 5/12 - 1/6 = ___

26. 2/5 - 2/10 = ___

27. 6/9 - 1/2 = ___

28. 3/4 - 3/6 = ___

14. 1 1/4 - 2/4 = ___

15. 1 1/7 - 2/7 = ___

16. 1 1/6 - 3/6 = ___

35. 1 1/8 - 1/4 = ___

36. 1 1/9 - 1/3 = ___

37. 1 1/5 - 2/3 = ___

9. 1 - 2/6 = ___

12. 3 - 1 1/4 = ___

10. 4 - 1/6 = ___

13. 6 - 1 2/10 = ___

11. 5 - 2/4 = ___

31. 8 4/8 - 2 1/3 = ___

39. 1 1/8 - 1/6 = ___

41. 1 1/5 - 4/10 = ___

43. 1 2/6 - 6/9 = ___

44. 4 1/5 - 2 2/7 = ___

38. 1 2/5 - 4/9 = ___

45. 6 1/6 - 3 2/4 = ___

46. 2 1/3 - 2/5 = ___

47. 7 3/6 - 3/4 = ___

40. 1 3/18 - 3/9 = ___

34. 5 3/5 - 3/9 = ___

42. 1 6/9 - 4/5 = ___ (Most Difficult)

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